Math 53: Multivariable Calculus **Sections 212, 213**

Worksheet for 2020-09-16

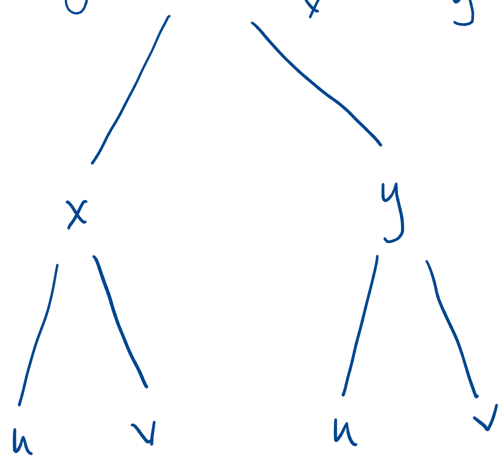
Problem 1. Let $f(x, y)$ and $g(u, v)$ be two functions, related by

$$g(u, v) = f(e^u + \sin v, e^u + \cos v).$$

Use the following values to calculate $g_u(0, 0)$ and $g_v(0, 0)$ (not all of the below values may be relevant!).

$f(0,0) = 3$	$g(0,0) = 6$	$f_x(0,0) = 4$	$f_y(0,0) = 8$
$f(1,2) = 6$	$g(1,2) = 3$	$f_x(1,2) = 2$	$f_y(1,2) = 5$

$$z = g(u,v) = f(\underbrace{e^u + \sin v}_{x}, \underbrace{e^u + \cos v}_{y})$$



$$g_u(u,v) = f_x(x,y)e^u + f_y(x,y)e^u$$

$$g_v(u,v) = f_x(x,y)\cos v + f_y(x,y)(-\sin v)$$

Note: If $u = v = 0$, then

$$x = e^0 + \sin 0 = 1$$

$$y = e^0 + \cos 0 = 2$$

So $g_u(0,0) = f_x(1,2)e^0 + f_y(1,2)e^0 = \boxed{7}$

$g_v(0,0) = f_x(1,2)\cos 0 + f_y(1,2)(-\sin 0) = \boxed{2}$

Problem 2. Consider the equation

$$x^7 - ax^6 + bx - 2 = 0. \quad (*).$$

If $(a, b) = (1, 2)$, then we have

$$x^7 - x^6 + 2x - 2 = 0$$

and you can check that $x = 1$ solves this equation. Now let's instead consider the equation

$$x^7 - 1.03x^6 + 2.06x - 2 = 0,$$

i.e. $(a, b) = (1.03, 2.06)$. Can you linearly approximate a solution for x to this equation? (Hint: use implicit differentiation to compute $\partial x/\partial a$ and $\partial x/\partial b$.)

$1^7 - 1^6 + 2 \cdot 1 - 2 = 0$ ✓

Write $x = f(a,b)$, where f implicitly defined by the equation $(*)$.

$$f(1.03, 2.06) \approx \underbrace{f(1,2)}_{=1} + 0.03 f_a(1,2) + 0.06 f_b(1,2)$$

$$\underbrace{x^7 - ax^6 + bx - 2}_{F(a,b,x)} = 0$$

$F_a(a,b,x) = -x^6 = -1$

$F_b(a,b,x) = x = 1$

$F_x(a,b,x) = 7x^6 - 6ax^5 + b = 3$

$$f_a(1,2) = -\frac{F_a(1,2,1)}{F_x(1,2,1)} = \frac{1}{3}$$

$$f_b(1,2) = -\frac{F_b(1,2,1)}{F_x(1,2,1)} = -\frac{1}{3}$$

So $f(1.03, 2.06)$

$\approx 1 + (0.03)\left(\frac{1}{3}\right) + (0.06)\left(-\frac{1}{3}\right)$

$= \boxed{0.99}$